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A multi-objective optimization model for dairy feeding management

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ABSTRACT

The allocation of feedstuff to intensively managed dairy cows to achieve different objectives is challenging due to the inherent complexity of the system and the combinatorial problem that has to be solved. Pareto-based multi-objective optimization approaches using evolutionary algorithms can help to address these challenges and show the trade-offs and synergies among various objectives. Here we present a framework for multi-objective optimization with the Differential Evolution (DE) algorithm applied to dairy feeding systems with grazing and concentrate supply to generate an approximation of the Pareto front. The available feed resources are located in different feeding areas, and the number of animals and groups of animals with similar feeding requirements are distributed across these areas for feeding purposes. To evaluate the DE algorithm, we performed two in-silico experiments to: (1) compare the solutions quality of single-objective DE with exact Linear Programming (LP) solutions, and (2) assess the influence of different stocking rates (number of cows/ha) on milk production, feed allocation and economic performance indicators. The DE solutions that minimize the feeding costs for different stocking rates (1.1–2.6 cows/ha) closely approached the solutions derived with LP, confirming the quality of the heuristic algorithm. The multi-objective model scenarios demonstrated that increasing stocking density would enhance milk production and gross margin per unit of area at largely unchanged productivity per animal by shifting the feed ration from roughage to a large proportion of supplementary concentrate feed. At low stocking rates solutions with high productivity and gross margin and a large proportion of roughage in the ration and limited supplementary feeding were identified. We conclude that the multi-objective optimization with a Pareto-based DE algorithm is highly effective to explore the interrelations among conflicting objectives and to find suitable solutions.

1. Introduction

In grassland-based dairy systems the stocking rate (units of animal per area), stocking method, and feed supplementation are important decision-making instruments that determine the effectiveness of systems, directly impacting feed intake, milk production and management efficiency in terms of labour productivity and economic viability. The variability in feed availability and quality may be high, in particular if a large proportion of intake is derived from pastures (Gregorini et al., 2017). The intensification of milk production in Uruguay has been based on a significant increase in the use of concentrates and conserved forage, while grazing remained unchanged (Fariña and Chilibroste, 2019). However, the viability of these practices and their productive and economic sustainability is debatable. While some specialists encourage the intensification of dairy systems, others question it. In particular, it is considered that managing a larger number of cows/ha can have an adverse impact on the dairy system, since infrastructure restrictions and animal welfare in this context are a challenge. The dairy industry is an important sector of the Uruguayan economy, and milk is mainly produced in grassland-based farming systems with supplementation (Chilibroste et al., 2015). The production of the dairy sector in Uruguay has been increasing in the last decades. Milk production has grown at rates of 5% per year (DIEA, 2017) while area reduced by 36% in the last thirty years, what represented an increase in productivity per hectare and per cow (Fariña and Chilibroste, 2019).

In a context of economic uncertainty, where the feed costs are high

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and the milk price is volatile, many producers are trying to harvest as much grass as possible, positioning pastures as the main feed for dairy cows, and relying on the use of supplements only when pasture dry matter scarce. In this scenario, to achieve a successful system, it is necessary to have control over the feed, the dairy herd and also a high level of management that guarantees an efficient allocation of feed resources to the dairy herd.

The allocation of feed resources to cows is a central decision problem in dairy farming systems. Animals differ in their feed requirements, in terms of both quantity and quality, based on their size, activity, growth and physiological status (lactation or pregnancy). Feed resources may be derived from various feeding areas, such as different pastures and the places where the cows receive supplements. In Uruguay these places for offering supplements are often a feed bunk where cows receive different mixes of concentrate and forages. These feed bunks are close to the milking parlour to minimize walking distance and machinery logistics. For feeding purposes the herd is usually split into groups of cows (typically based on parity and/or the level of production), and each group is distributed to these feeding areas. The composition of each group remains unchanged for a certain period of time, typically one month. After that period new groups of cows are defined, so the distribution process into the feeding areas stars over. This procedure is repeated throughout the year. In Uruguay, this feed allocation process is usually carried out based on the experience and intuition, and even traditions of the producers, following management rules.

The problem we are dealing with in this study is to determine how to group and distribute the cows into the feeding areas throughout the year. This problem is difficult to solve when the problem size increases (there are many possible combinations for grouping and distribution) and/or when resources are scarce, so addressing the problem by studying techniques that serve as support in decision making can be of great help to the dairy sector. In this sense, optimization techniques have been widely used and considered as very useful for agricultural models (Weintraub and Romero, 2006). One of the first mathematical programming applications in this area was reported by Waugh (1951), who used linear programming models to determine the minimum cost of the livestock ration. The problem of determining the most cost-effective combination of forage species was addressed by Neal et al. (2007), who used a linear programming model. Dean et al. (1972) used production functions and linear programming models to develop a computer system capable of providing feeding programs that optimized feeding dairy cattle. In particular, they analyzed the possibilities of increasing the efficiency and profitability of milk production per cow. Ridler et al. (2001) focused on an economic linear programming model to maximize the farm profitability. Later, dynamic programming, nonlinear programming, and nonlinear optimization models were also used to evaluate dairy production systems (Kalantari et al., 2010; Doole et al., 2012, 2013; Doole and Romera, 2013).

However, for situations where multiple objectives are involved and the potential trade-offs and synergies among these objectives should be explored, heuristic approaches may be more suitable (Groot and Rossing, 2011). These methods, unlike exact methods, do not guarantee finding global optimal solutions, but they have been used to obtain good quality approximate solutions in a reasonable execution time. Evolutionary algorithms (Bäck, 1996), inspired by Darwin's theory of evolution, are optimization and search methods based on a heuristic approach that belong to the artificial intelligence branch. They are computational models simulating the crossover, mutation natural selection of genotypes. Their operation is based on the formation of a set of genotypes that represent possible solutions, which are mixed and compete with each other. The genotypes resulting in the more desirable performance (phenotype) are considered to be the fittest that prevail over time, due to a selection process that occurs in each generation. The implementation of these models consists of determining the parameters of the problem, coding them in a chromosome (genotype) format and applying operators of evolution. These algorithms have proven to be flexible and robust methods for effectively solve complex, multi-objective optimization problems, and have been applied in different contexts, like agricultural systems, land-use allocation and industrial systems (Deb and Kalyanmoy, 2001; Coello et al., 2006; Groot et al., 2007, 2010; Gong et al., 2013; Memmah et al., 2015; Han et al., 2015, 2016; Gong et al., 2018; Han et al., 2019).

The general objective of this study is to develop a multi-objective and multi-period model to allocate the available feed resources to a dairy herd. We also want to identify and analyze the potential trade-offs between the different objectives. The allocation is done based on a division into groups of cows and distributing the groups to the different feeding areas (considering a multi-period approach of 12 months) in order to satisfy the multiple objectives of maximizing: milk production, margin over feeding cost, and herbage intake; and minimizing: cost of the diet and supplement intake. To solve the model, we used an multi-objective evolutionary optimization algorithm to generate an approximation of the Pareto front. In addition to developing, testing and validating the usefulness of the methodology in the context of the dairy feeding systems problem, an additional aim was to apply it to typical Uruguayan dairy farming systems. The contribution of this work lies in the proposal of a powerful and novel method for the dairy sector, which can be useful to determine different strategies for animal grouping and food resource allocation considering multiple objectives.

2. Materials and methods

2.1. Herd management, milk production and feed allocation

The optimization problem entails reaching the multiple objectives of productivity, profitability and efficiency of dairy production simultaneously through an optimal allocation of available feed resources to the dairy herd. The objectives of the optimization are to maximize milk production (liter cow^{-1} day^{-1}), to maximize the gross margin over the feeding costs (USD cow^{-1} day^{-1}), to maximize the herbage intake from pastures (kg of DM cow^{-1} day^{-1}), to minimize the costs of the ration (USD cow^{-1} day^{-1}) and to minimize the intake of supplements (kg of DM cow^{-1} day^{-1}). The feed sources are various pastures and supplements that are located in different feeding areas and are characterized by their amount, nutritive value and distance from the milking parlour. The cow herd consists of different types of animals differentiated on the basis of body weight, milk production level, parity and lactation stage. The herd can be divided into groups to facilitate the handling by the farm manager. For each cow group the number of visits to the various feeding areas is determined for a certain period of time, after which the groups may be redefined and reallocated to feed resources. The feed resource allocation to the cow groups is conducted for one whole year, divided into twelve monthly periods. The lactating cows are milked and fed twice per day, so the feed area allocation is performed twice per day. Then, for each group, the number of allocations for a specific period is calculated as the number of days of that period multiplied by two, resulting in 56 (February) to 62 (e.g., March) assignments per period. Future versions of the model will be extended so that the number of periods and days per period can be entered as input, allowing the user to define the temporal scope of the execution.

The input data for the model are the number of periods of time, number of groups of cows, types of cows, number of cows for each type, characteristics of the feeds in the different areas, and prices of feeds and milk. Cows with similar characteristics were considered as cows of the same type. Each type of cow is differentiated by the following attributes: body weight (*bw*, kg), genetic potential (*gp*, liters of milk in 305 days), lactation days (*ld*) or lactation weeks (*lw*) and fat (*g*) and protein (*p*) content in milk. We considered animals with varying body weight, genetic potential and number of weeks in lactation, while we fixed (without loss of generality) the other parameters to the following values: $g = 3.6\%$, $p = 3.1\%$. These values were based on typical

Uruguayan farms values (Fariña and Chilibroste, 2019). Per cow type energy requirements are calculated and feed intake capacity is established. The current model considers that each type of cow (identified by body weight, genetic potential and lactation days) remains unchanged throughout the year, therefore the lactation curve is the same. Future versions of the model will be extended so each type of cow can be discriminated for each period. Feedstuff were differentiated into pastures and supplements. Pastures differ in dry matter (DM) productivity (herbage mass in Mg DM per ha), energy content (expressed in net energy for lactation in MCal per kg DM) and costs (USD per Mg DM). Pastures were considered as a finite resource and DM availability and consumption during a period were used to calculate the herbage mass in the next period. Supplements were a combination of conserved forage and concentrates that differed in availability, energy content and costs.

Milk production (*M*; kg per day) was derived from the amount of energy available for lactation, which was calculated as the difference between the energy in consumed feed (*cInt*; MCal) and the requirements for maintenance (*bReq*; MCal), movement (*mReq*; MCal) and grazing (*gReq*; MCal) (Eq. (1)). Requirement calculations were based on the nutrient requirements of dairy cattle as published by the (U.S.) National Research Council (2001). The amount of net energy needed to produce one kg of milk (*eM*; MCal kg^{-1}) depends on the fat and protein content of the milk (Eq. (2)).

$$M = \frac{cInt - bReq - mReq - gReq}{e_M} \tag{1}$$

$$eM = 0.0929 \times g + 0.0547 \times p + 0.192 \tag{2}$$

Maintenance requirements depend directly on the metabolic weight ($mw = bw^{0.75}$; see Eq. (3)). Movement requirement is related to the walking distance and *bw* (Eq. (4)), while energy required for grazing is assumed to be proportional to the maintenance requirement (Eq. (5)).

$$bReq = f_r \times mw \tag{3}$$

$$mReq = 2 \times f_m \times d \times bw \tag{4}$$

$$gReq = f_g \times bReq \tag{5}$$

where:f_r = proportionality constant for energy requirement for maintenance (0.08 MCal $kg^{-0.75}$);f_m = proportionality constant for energy requirement for movement (0.00045 MCal km^{-1});d = distance between the milking parlour and feeding area (km);f_g = proportionality constant for energy requirement for grazing (0.15 MCal kg^{-1}).

The feed intake capacity (*C*; expressed in kg DM day^{-1}) defining the upper limit of feed consumption per animal per day is proportional to potential milk production and metabolic weight (Eq. (6)).

$$C = (f_p \times P_M + f_i \times mw) \times (1 - e^{-0.192 \times (lw + 3.67)}) \tag{6}$$

where:P_M = potential milk production (liter day^{-1});f_p = proportionality constant for potential milk production (0.372 kg DM $liter^{-1}$ milk);f_i = proportionality constant relating intake to body weight (0.0968 kg DM kg^{-1} metabolic weight).

The gross margin over feeding cost (*Ma*) was calculated as the revenues from milk production minus feeding costs. The model was built for an empty/un-pregnant dairy cow on neutral energy balance. In further developments of the model changes in body condition score or the actual energy balance of the animal will be included, as well as pregnancy.

2.2. Multi-objective optimization with an evolutionary algorithm

The multi-objective optimization problem can be generally formulated as in Eqs. 7–9, where $U_1(x)$, …, $U_k(x)$ are the objective functions that are simultaneously maximized or minimized, and (x_1, …, x_n) are the decision variables (Table 1).

Table 1
Decision variables, objective functions and constraints description.

Acronym	Description
Decision variables	
x	Number of cows of each type in each group
y	Number of times each group is assigned to each feeding area
w	Total intake of DM for each group in each feeding area
v	Available feed in each feeding area
Objective functions	
MP	Milk production
Ma	Gross margin over feeding costs
HI	Total herbage intake
SI	Total supplement intake
C	Feeding costs
Constraints	
RAC	feed allocation constraints
AFC	feed availability constraints
BC	biological constraints

$$MinU(x) = (U_1(x), U_2(x), ..., U_k(x))^T \tag{7}$$

$$x = (x_1, x_2, ..., x_n)^T \tag{8}$$

Subject to i constraints:

$$g_i(x) \le h_i \tag{9}$$

The resource allocation model was formulated as a mathematical programming model, and the full description can be found in Appendix A.

The decision variables were implemented as an array of integers and represented, for each period, the number of cows of each type in each group and the number of times each group is assigned to each feeding area (cf. Goldberg, 1989; Talbi, 2009; Notte, 2014; Notte et al., 2016). The number of groups, cow types and feeding areas were defined as input parameters, and the size of the array of decision variables was determined from those values. A major advantage of this encoding is its simplicity, but it is possible to generate infeasible solutions due to the randomness that is used in the evolutionary operators. In the mutation operator, the value of a variable is modified randomly. In the crossing operator, two solutions are randomly selected and two new solutions are generated from their crossing (see below). Because of this, solutions can be created with an incorrect number of cows, either violating the size of the herd, the number of cows of each type or the number of cows in each group. Other possible causes for infeasibility are the incorrect number of times each group goes to a feeding area or solutions that do not satisfy the biological restrictions. To ensure feasibility, when an infeasible solution was detected the recombination process was repeated until a feasible solution was found.

A Pareto-based multi-objective variant of the evolutionary algorithm on Differential Evolution (DE; Storn and Price, 1995; Groot et al., 2007, 2012, 2018) was used to explore the solution space defined by the constraints of the optimization problem. The DE algorithm requires only three parameters CR (defining the crossover probability), F (scaling factor of the difference of two individuals) and N (population size) to generate the evolutionary process. The decision variables are represented as a genotype consisting of a multi-dimensional vector $p = (a_1, \dots, a_z)^T$ of z alleles. Each allele a_i is initialized as $a_{i,\,0}$ by assigning a random number within the range allowed for individual decision variables. The genotypes are part of a population that is iteratively improved. As described by Groot et al. (2012), a new generation $t + 1$ is created by applying mutation and selection operators on each of the individuals in the population *P* of the current generation *t*. The first step of the reproduction process is generation of a trial population *P′* that contains a counterpart for each individual in the parent population *P*, produced by parameterized uniform crossover of a parent vector and a mutation vector. The mutation vector is derived from three

Create a random initial population
for i = 1 to N **do**
  for j = 1 to Z **do** // *Z = Number of alleles*
    $p_{j,i}^{t=0} = p_j^{min} + rand_j[0, 1] * (p_j^{max} - p_j^{min})$ // *t = generation*
  end for
end for
Evaluate fitness function for each individual of the population
for i = 1 to N **do**
  $U(p_i^{t=0})$
end for
Trial vector generator
for t = 1 to MaxGeneration **do**
  for i = 1 to N **do**
    Select randomly $c_1, c_2, c_3 \in [1, N], c_1 \neq c_2 \neq c_3 \neq i$
    Mutation and Crossover Process
    $jrand = randInt[1 : Z]$
    for j = 1 to Z **do**
      if $rand[0, 1] < CR$ **then**
        $v_{i,j}^{t+1} = p_{i,c_1}^t + F * (p_{i,c_2}^t - p_{i,c_3}^t)$
      else
        $v_{i,j}^{t+1} = p_{i,j}^t$
      end if
    end for
    Selection
    if $U(v_{i,j}^{t+1}) \leq U(p_i^t)$ **then**
      $p_i^{t+1} = v_{i,j}^{t+1}$
    else
      $p_i^{t+1} = p_i^t$
    end if
  end for
end for

Algorithm 1. DE algorithm.

mutually different competitors c_1, c_2 and c_3 that are randomly selected from the population P in the current generation t. A trial genotype p_{t+1}' replaces p_t if it outperforms the parent genotype. Here, better performance is interpreted as a better Pareto ranking or a location in a less crowded area of the search space than the parent genotype.

The pseudocode of the DE algorithm is presented in Algorithm 1. Parameters used for the DE algorithm were cross-over amplitude (F = 0.7), cross-over probability (CR = 0.85), number of genotypes in the population (N = 1000) and the number of iterations to improve the population (T = 1000). The DE algorithm used 156 decision variables. See Groot et al. (2007, 2010, 2012) for more details on the Pareto ranking procedure and the evolutionary algorithm.

2.3. Computational experiments

We conducted two computational experiments (1) to assess the performance of the evolutionary algorithm and (2) to determine the influence of different stocking rates (the total number of cows considered in the system divided by the number of hectares) on the trade-offs and synergies among the objectives. Experiment (1) compared the multi-objective optimization results, in particular for the objectives to minimize the cost of the ration and maximize the gross margin over feeding cost, with those of a mono-objective linear programming (LP) model used in an ongoing project using on-farm collected data. In the LP model milk production levels, characteristics of the herd and the feeding options were defined as inputs, while the feeding cost was defined as the objective to minimize. The gross margin over feeding cost is calculated from the price of milk and the costs of the system. The same biological constraints are used in the LP model as in the mathematical model, presented in Appendix A. The LP optimization was conducted for two milk production levels, of 25 and 30 l cow^{-1} day^{-1}, and four stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1}, where 128, 185, 247 and 309 cows in the system were considered respectively. The LP results were compared with the results of four DE optimization runs (one for each stocking rate), of which three solutions were obtained with milk production levels of ca. 25 l cow^{-1} day^{-1}, and three others ca. 30 l cow^{-1} day^{-1}. For each stocking rate and milk production level, the values of cost of the ration and margin over feeding costs obtained from the LP model were compared with the average values of the three solutions obtained from the DE optimization run. In this experiment we use the same input data for both models, except that milk production is an input in the LP model and an objective in the multi-objective model. In Experiment (2) we compared the results of the multi-objective DE optimization for the five objectives at stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1}. In both Experiments (1) and (2) we considered one cow type with body weight bw = 580 kg, genetic potential gp = 8500 l of milk in 305 days and 20 weeks of lactation. We included eleven feeding areas or zones (Z1, Z2, …, Z11). Nine of these corresponded to equally-sized pastures (Z1, …, Z9) comprising an area of 117 ha. The pastures used were prairies, oats, ryegrass and sorghum. Additionally, two corresponded to feeding areas where supplements were supplied (Z10 and Z11). Feed characteristics (energy content, availability throughout the year, cost and the distance (km) between the milking parlour and pastures) are summarized in Table 2. The quality of the pasture was defined by the energy content. In this work, we did not represent pasture growth, instead we decided to express the amount of pasture available in each period, which is an input of the model based on pasture growth rate. We did not consider a specific price for the pasture either, instead we considered an average cost of 302 USD per ha to produce the selected pastures during the twelve monthly periods (value obtained from the LP model). The cost of the diet is presented per cow and per day, so the total cost to produce the pastures was evenly distributed per cow and per day. The prices of supplement Z10 and Z11 (mixes of concentrate and forages) were defined as a 60% and 80% of the milk revenue per kilogram of DM, respectively. For these experiments we considered the milk revenue as 0.30 USD per liter. Milk and concentrates prices are based on last years information received by Uruguayan dairy farmers (IFCN, 2019).

3. Results

3.1. Experiment (1) - performance of the DE algorithm

In Experiment (1) the LP model attained the lowest feed costs (Fig. 1a) as the exact solution of the single objective optimization

Table 2
Characteristics of the feeding areas.

Activity	Description	ED (Mcal ENl/kg DM)	Distance (km)	Availability (kg DM)	Price (%)
Z1	Pasture	1.4	0.5	72,522	–
Z2	Pasture	1.4	1.5	107,227	–
Z3	Pasture	1.4	2.5	102,026	–
Z4	Pasture	1.4	0.5	89,054	–
Z5	Pasture	1.4	1.5	115,322	–
Z6	Pasture	1.4	2.5	112,608	–
Z7	Pasture	1.4	0.5	27,502	–
Z8	Pasture	1.4	1.5	49,974	–
Z9	Pasture	1.4	2.5	81,357	–
Z10	Mix 1	1.5	0	∞	60
Z11	Mix 2	1.7	0	∞	80

Notes: Activity = food activity, Description = description of the food activity, Mix = mix of concentrate and forages, ED = energy density measured like the net energy megacalories per lactation per kilogram of dry matter, Distance = distance to de milking parlour, Availability = availability of the activity throughout the year, Price = price measured like a percentage of the milk price per kilogram of DM.

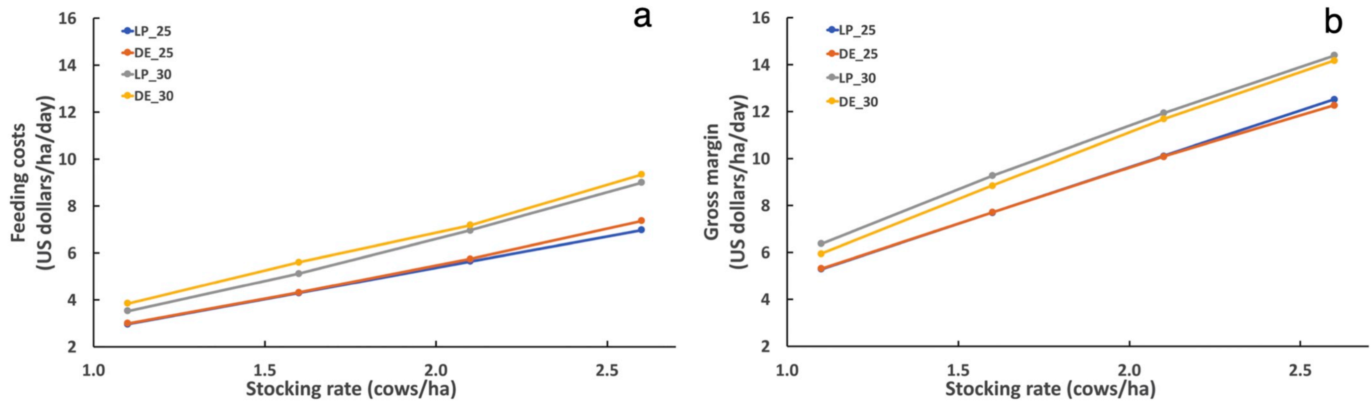

Fig. 1. Optimized feeding costs (a) and gross margin over feeding costs (b) as related to stocking rate for dairy cows feeding systems derived from Linear Programming (LP) and Differential Evolution (DE) algorithms, at different production levels of 25 (LP_25 and DE_25) and 30 (LP_30 and DE_30) l of milk per day.

problem minimizing feeding costs. The cost values reached by the LP model were closely approached by the DE algorithm (Fig. 1a), especially for those solutions of 25 l cow^{-1} day^{-1}. When 25 l cow^{-1} day^{-1} were considered, for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1} the values obtained by the DE algorithm were 1.69%, 0.23%, 1.24% and 6.17% higher than the values obtained by the LP model respectively. When 30 l cow^{-1} day^{-1} were considered, for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1} the values obtained by the DE algorithm were 7.95%, 9.37%, 3.59% and 3.45% higher than the values obtained by the LP model respectively. Margin values behaved in a similar way to those of cost (Fig. 1b). When 25 l cow^{-1} day^{-1} were considered, for the stocking rates of 1.1 and 1.6 cows ha^{-1} the values obtained for both methods were the same, while for stocking rates of 2.1 and 2.6 cows ha^{-1} the values obtained by the LP model were 0.2% and 2.1% higher than the values obtained by the DE algorithm respectively. Let us remember that the values of the LP model were compared with the average of three values of the DE algorithm, therefore when the average is a little greater than 25 or 30 (depending on the case) the difference between the margins obtained by both techniques is smaller than expected. When 30 l cow^{-1} day^{-1} were considered, for the stocking rates of 1.1, 1.6, 2.1 and 2.6 cows ha^{-1} the values obtained by the LP model were 7.05%, 4.86%, 2.14% and 1.55% higher than the values obtained by the DE algorithm respectively.

3.2. Experiment (2) - influence of different stocking rates

The results of the multi-objective DE optimization with the five objectives for stocking rates of 1.1 and 2.1 cows/ha from Experiment (2) are presented in Fig. 2. In Table 3 the best attainable solutions for the different objectives are presented for the four stocking rates.

For both stocking rates of 1.1 and 2.1 cows/ha, a clear trade-off was visible between milk production per cow and feeding costs (Fig. 2a) due to an increase in the amount of supplements fed (Fig. 2d) and a reduction of herbage consumption (Fig. 2b) at higher milk production levels. This is due to the existence of a trade-off between herbage consumption and supplement consumption, when one increases the other decreases (Fig. 2f). Regardless of the stocking rate, the highest level of milk production was achieved with a diet based on supplements only (Table 3).

With increasing cow productivity, the margin over feeding costs increased (Fig. 2g). For the stocking rate of 1.1 cows/ha, the greatest margin was reached at feeding costs of less than 3 USD per day, while for the stocking rate of 2.1 cow/ha, the greatest margin was reached with costs of almost 3.5 USD per day (Fig. 2h; Table 3). For the stocking rate of 1.1 cows/ha, the highest values for gross margin over feeding costs were obtained when high levels of herbage intake were combined with intermediate supplementary feeding of between 7 and 9 kg cow^{-1} day^{-1} (Fig. 2i and j; Table 3). In contrast, at higher stocking rate of 2.1 cows/ha, the highest margin over feeding costs was reached at higher supplementary feeding and lower herbage intake (Fig. 2i and j; Table 3). By increasing the stocking rate from 1.1 to 2.1 cows/ha, the number of cows in the system is almost double, but grassland resources remain unchanged. The carrying capacity of the farm is exceeded and external land/feed is used to make this up.

From Fig. 2, when comparing both stocking rates, we see that in some cases the solutions for the stocking rate of 2.1 cow/ha almost completely overlap with those of 1.1 cow/ha (Fig. 2a, g and h). In these cases the range of solutions obtained by the stocking rate of 1.1 cows/ha is more diverse than the one obtained by the stocking rate of 2.1 cows/ha. There are also cases where the overlap is smaller (Fig. 2b, d, f, i and j) or almost imperceptible (Fig. 2c and e).

From Table 3, results showed that when the stocking rate increases from 1.1 to 2.6 (136%), the margin over the feeding cost per hectare also increases for any objective. Particularly, for the objective of maximizing the milk production the increase was 162%, while for the objective of maximizing the herbage intake the margin per hectare increased 104%. At higher stocking rate, the pasture production is used by more cows requiring increased supplement offer. While thus both the feeding cost per hectare and the milk production per hectare increased, the result was that the margins over feeding costs per hectare were greater at higher stocking rates. When maximizing the margin per cow, increasing stocking rate decreased average herbage intake up to 50%, while the supplement intake increased up to 59% per cow. At the same time, the productivity per hectare increased up to 163%, and the margin per unit of area increased up to 141% (Table 3).

For the objective of maximizing the herbage intake, by increasing the stocking rate the herbage intake per cow was reduced from 12.9 to 6.7 kg cow^{-1} day^{-1} because the pasture was shared among more cows. The consumption per hectare increased from 5175 to 6386 kg ha^{-1} year^{-1}, which shows a limitation of the DE algorithm when maximizing pasture consumption for a low stocking rate (1.1 cows/ha). For higher stocking rates the DE algorithm correctly solves the maximization. Maximum margin was achieved at less-than-maximum levels of milk production, so it is not necessary to reach the highest levels of milk production to maximize profit.

In Table 3 we see that the milk production values of the solutions that maximize the milk production are between 11% and 21% (depending on the stocking rate) higher than the milk production values of the solutions that maximize the margin. Maximum margin is achieved at less-than-maximum levels of herbage intake. Maximum margin over feeding costs are between 34% and 66% higher, depending on the stocking rate compared to margin over feeding costs associated with maximized herbage consumption. Unlike the previous cases, the largest percentage difference occurs when the stocking rate is high.

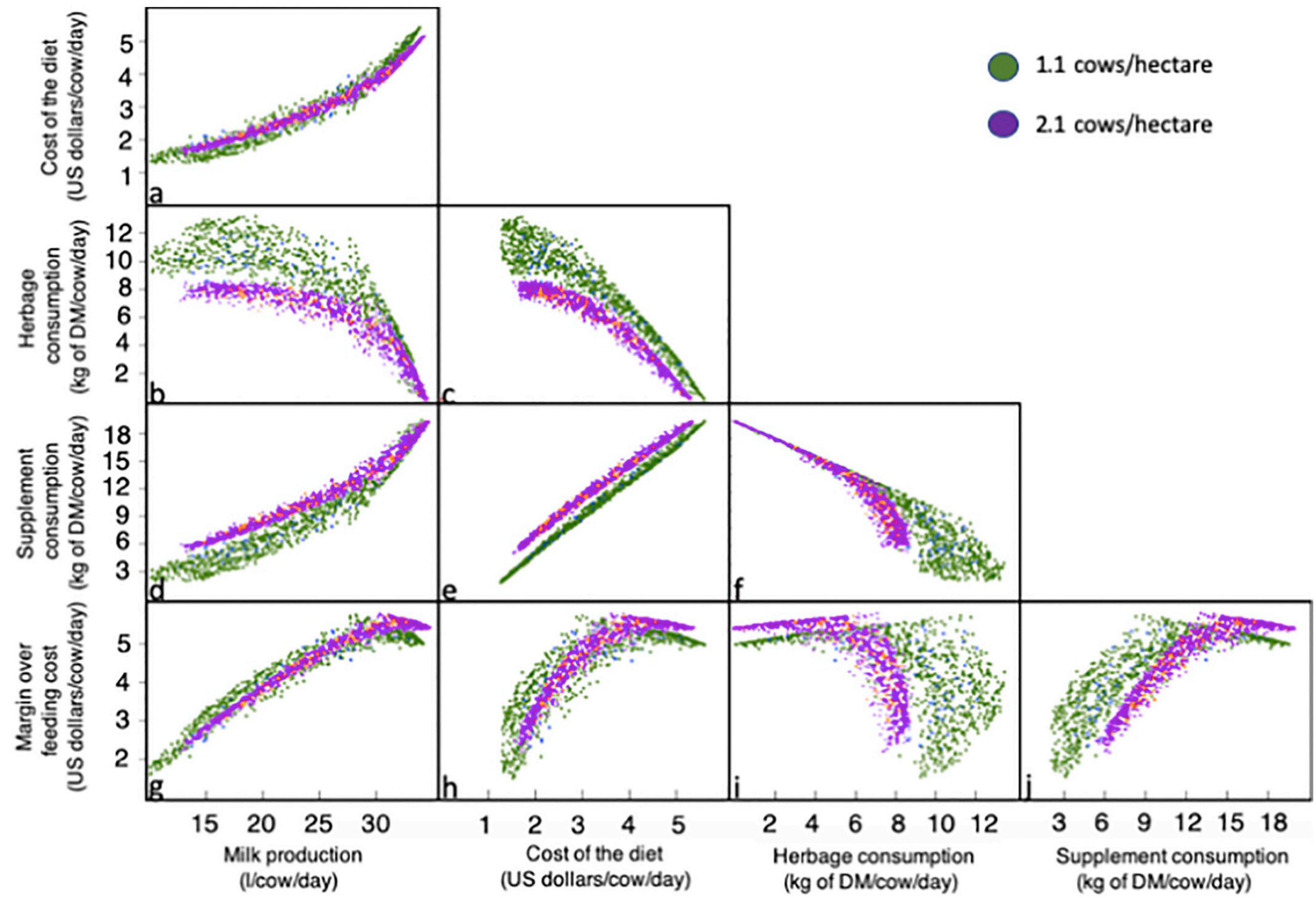

Fig. 2. Relationship between the dairy system performance indicators as represented by Pareto frontiers after multi-objective optimization. Each dot represents a way to do the food resource allocation to the dairy herd. The green dots represent the solutions obtained using a stocking rate of 1.1 cow/ha, while the violet dots represent the solutions obtained using a stocking rate of 2.1 cows/ha.

3.3. Decision variables analysis - distribution of cows

The results presented above were achieved from the distribution of the cows among the different feeding options (pastures or supplements) throughout the year, and the values of the decision variables describe how to perform that distribution. In particular, these values indicate, for each period of time (month), how many times the herd must be assigned to each feeding option (or feeding zone).

For each of the four runs performed in Experiment (1) (one for each stocking rate), three solutions with the highest milk production, margin over feeding cost and pasture consumption were obtained. For each solution, the number of times the herd must be assigned to pastures (Z1 to Z9) and supplements (Z10 and Z11) is summarized and presented in Table 4. Considering that cows are fed twice a day and the year has 365 days, there are 730 annual assignments.

For the stocking rate of 1.1 cows/ha, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, only between 3.0% and 6.2% of the assignments were on pastures, while most of them were on supplementation, which means that supplements were the main part of the diet; in the solutions with the highest margin over feeding cost, the assignments were balanced between pastures and supplements; in the solutions with the highest herbage intake, most of the assignments were on pastures. For the stocking rate of 1.6 cows/ha, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, few assignments were on pastures, while most of them were on supplementation; in the solutions with the highest margin over feeding cost, between 32.3% and 53.0% of the assignments were on pastures, while between 47.0% and 67.7% were on supplementation; in the solutions with the highest herbage intake, between 62.9% and 71.8% of the assignments were on pastures and the rest were on supplementation.

For the stocking rate of 2.1 cows/ha, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, at most 1.5% of the assignments were on pastures; in the solutions with the highest margin over feeding cost, between 34.8% and 39.1% of the assignments were on pastures, while the rest of them were on supplementation; in the solutions with the highest herbage intake, between 62.5% and 65.1% of the assignments were on pastures, while between 34.9% and 37.5% were on supplementation.

Finally, for the stocking rate of 2.6 cows/ha, the DE algorithm proposed the following distribution of cows: in the solutions with the highest milk production, the situation was similar to the scenario with a stocking rate of 2.1 cows/ha (at most 1.5% of the assignments were on pastures); in the solutions with the highest margin over feeding cost, between 26.4% and 33.5% of the assignments were on pastures, while between 66.4% and 73.6% were on supplementation; in the solutions with the highest herbage intake, only between 52.7% and 68.3% of the assignments were on pastures, while between 31.7% and 47.3% were on supplementation.

As the stocking rate increases, the availability of pastures per cow decreases, so it is necessary to add more supplements in the diet. This is clearly reflected in the solutions with the highest pasture consumption.

Table 3
Best attainable values at different stocking rates for the five objectives of the multi-objective optimization with DE. The objective with the highest value is indicated in bold font.

Objective	Stocking rate			
	1.1 cows/ha	1.6 cows/ha	2.1 cows/ha	2.6 cows/ha
Milk production:	**33.9**	**34.3**	**34.7**	**34.7**
Gross margin:	4.84	5.12	5.29	5.37
Feeding costs:	5.32	5.16	5.13	5.02
Herbage consumption:	0.02	0.07	0.04	0.15
Supplement consumption:	19.8	19.7	19.7	19.6
Milk production per hectare:	37.3	54.8	72.9	90.1
Gross margin per hectare:	5.32	8.19	11.1	13.9
Milk production:	27.9	30.1	30.1	31.1
Gross margin:	**5.55**	**5.57**	**5.61**	**5.65**
Feeding costs:	2.83	3.46	3.42	3.67
Herbage consumption:	10.8	7.30	6.79	5.48
Supplement consumption:	8.98	12.5	12.9	14.3
Milk production per hectare:	30.8	48.2	63.2	80.9
Gross margin per hectare:	6.10	8.91	11.8	14.7
Milk production:	10.4	9.87	12.2	12.0
Gross margin:	1.92	1.80	2.15	1.97
Feeding costs:	**1.19**	**1.16**	**1.49**	**1.63**
Herbage consumption:	10.7	9.20	7.40	6.26
Supplement consumption:	2.05	3.05	5.56	6.37
Milk production per hectare:	11.4	15.8	25.5	31.3
Gross margin per hectare:	2.11	2.88	4.51	5.12
Milk production:	19.2	19.5	17.6	19.4
Gross margin:	4.14	4.01	3.38	3.57
Feeding costs:	1.61	1.85	1.89	2.25
Herbage consumption:	**12.9**	**10.4**	**8.36**	**6.73**
Supplement consumption:	3.96	6.23	6.90	9.05
Milk production per hectare:	21.1	31.3	36.9	50.5
Gross margin per hectare:	4.55	6.41	7.09	9.28
Milk production:	10.4	9.87	13.2	12.0
Gross margin:	1.92	1.80	2.36	1.97
Feeding costs:	1.19	1.16	1.58	1.63
Herbage consumption:	10.7	9.20	7.88	6.26
Supplement consumption:	**2.05**	**3.05**	**5.40**	**6.37**
Milk production per hectare:	11.4	15.8	27.6	31.3
Gross margin per hectare:	2.11	2.88	4.95	5.12

Notes: Cows per hectare = cows/ha, Milk Production = l cow^{-1} day^{-1}, Gross margin = USD cow^{-1} day^{-1}, Feeding cost = USD cow^{-1} day^{-1}, Herbage consumption = kg of DM cow^{-1} day^{-1}, Supplement consumption = kg of DM cow^{-1} day^{-1}, Milk production per hectare = l ha^{-1} day^{-1}, Gross margin per hectare = USD ha^{-1} day^{-1}.

4. Discussion

The DE algorithm was effective in solving the mono-objective optimization problem and allowed exploration of the relations among objectives for the multi-objective optimization problem. The multi-objective model scenarios demonstrated that increasing stocking density would enhance milk production per unit of area and gross margin per unit of area, while the feed ration would shift from roughage to a large proportion of supplementary concentrate feed. Increasing stocking rates caused big differences in the results.

The algorithm generated a wide range of solutions that showed the trade-offs among the objectives, reaching extreme values close to the

Table 4
Number of times the herd must be assigned to pastures (P) or supplements (S).

Objective	Stocking rate							
	1.1 cows/ha		1.6 cows/ha		2.1 cows/ha		2.6 cows/ha	
	P	S	P	S	P	S	P	S
Milk production: (l cow^{-1} day^{-1})	41	689	55	675	11	719	11	719
	45	685	46	684	4	726	6	724
	22	708	9	721	0	730	4	726
Gross margin: (USD cow^{-1} day^{-1})	412	318	258	472	286	444	202	528
	393	337	236	494	254	476	193	537
	431	299	387	343	274	456	245	485
Herbage consumption: (kg of DM cow^{-1} day^{-1})	649	81	524	206	475	255	479	251
	591	139	459	271	470	260	385	345
	597	133	511	219	456	274	499	231

values obtained by the LP model in the minimization of costs and maximization of the margin over the feeding cost. This closeness of the values can be seen in Fig. 1a, where the results of the costs are shown, and in Fig. 1b where the results of the margins are shown. In Experiment (1) we verified that the lower feeding costs obtained by the DE algorithm were within 0.23% and 1.24% higher than those obtained by the LP model (optimal), while the highest margins obtained by the DE algorithm were less than 2.1% lower than those obtained by the LP model, which demonstrates the good performance achieved by the DE algorithm. In Experiment (2) we found that increasing the stocking rate resulted in higher milk production and margin over feeding cost. As the number of cows/ha increased, the potential herbage intake per cow exceeded herbage availability, and therefore the supplement intake and the cost of the diet were higher. At low stocking rates, solutions with high productivity and gross margin were identified, whose diet was based on a high herbage intake and limited supplement intake.

In most multiobjective problems it is not easy or it is not possible to obtain an exact description of the Pareto front set because it can cover a very large or infinite number of points. Although in theory it is possible to find these points exactly, it is computationally difficult and expensive (Ehrgott and Wiecek, 2005). To the best of our knowledge, the model used in this work is the first one that uses the DE algorithm to analyze the feed resource allocation to a dairy herd and that considers the grouping of cows and has different production potential. The solutions obtained by the DE algorithm were validated and considered representative of the system by an expert in the dairy sector. The effect of the stocking rate has been widely studied, but the novelty of the presented model is that its behavior can be studied taking into account different system conditions and without losing focus on the proposed objectives.

From the results obtained, the observed trends in productivity and profitability per animal and per unit of area with increasing stocking density are in line with local (Ortega et al., 2018; Fariña and Chilibroste, 2019) and international (Baudracco et al., 2011) research. For the objective of maximizing the milk production, the herbage intake values were significantly lower than the values obtained when maximizing the herbage intake (for any of the stocking rates considered) and the supplement intake values were significantly higher than the values obtained when minimizing the supplement intake. From the logical point of view of the model, the high intake of supplements in solutions that maximize milk production is mainly due to the fact that the supplements provide a higher energy density than pastures, and milk production is directly related to the energy acquired from the feed consumption. However, we have observed that for both objectives, minimizing the intake of supplement and minimizing the feeding cost the model converged to very low production levels (within 28.8% and 38% of maximum production) according to the breed and animal live weight used in the experiments. For the objective of maximizing the margin over feeding cost, by increasing the stocking rate from 1.1 to 2.1 cows/ha, herbage intake levels cannot be maintained because the available pastures do not produce enough dry matter to provide a sufficient amount of feed. Due to this, the model proposed feeding strategies with higher consumption of supplements and lower herbage consumption. This strategy directly impacts on the feeding cost, since the relationship with it and the supplement consumption is almost linear. Minimizing the feeding cost seems reasonable or even necessary in many dairy systems, but this does not ensure greater profitability. Although each system has its peculiarities and limitations, from this work we can appreciate that it is possible to optimize profitability through a high milk production combined with a controlled cost of feeding, which is achieved when the diet is balanced in herbage and supplements consumption.

In dairy systems it is increasingly important and necessary to use models that represent the systems for several reasons: (i) in order to deal with the complexity they present, which includes many components and variables, (ii) to address various objectives, limitations and opportunities that are relevant to the producer, and (iii) to anticipate and respond to the constant change that dynamic systems such as the dairy system presents. These types of tools, which allow incorporating and combining different indicators or objectives, generate alternatives that allow opportunities for discussion and analysis from different points of view. For example, from the results presented, it can be clearly seen that for some systems, by maximizing milk production or minimizing the cost of the diet, the economic benefit will not necessarily increase. Also, this type of tools is very powerful to simulate different production scenarios and analyze the different variants to consider in those cases. For example, scenarios that incorporate adverse climatic factors can be simulated, and therefore analyze what would happen to a productive system that could eventually have a lower amount of grass in a given period of the year. In this particular work, by exploring tradeoffs among objectives, the DE algorithm was used to show different options for decision makers on how to do the food resource allocation, letting them to choose the one that fits better for their productive system. Through this tool it is also possible to explore different alternatives to those commonly used by producers. In turn, it allows to analyze the behavior of different production systems when some parameters are modified. Particularly from Fig. 2 it is possible to evaluate how the responses and interactions in the dairy system can change as stocking rates increase.

As future work, it would be useful to extend the resource allocation model and to perform an in-depth computational performance analysis. This could include performing many executions with different algorithm parameter values in order to find the best configurations. Also, the DE algorithm could be executed with a larger number of iterations, to evaluate the results quality considering different indicators as well as the computational costs. To carry out this study, reference points will be generated from the results obtained by running different algorithms. In particular, some of the algorithms that have shown good performance in solving discrete multi-objective optimization problems are NSGAII, SPEA and GDE (Zitzler and Thiele, 1998; Zitzler and Thiele, 1999; Zitzler et al., 2000). Also more experiments to analyze the decision variables and the composition of the solutions can be considered, including more scenarios with several groups of different types of cows.

5. Conclusions

The DE algorithm proved to be effective in representing the problem presented in this work; it had the capacity to handle the different constraints of the dairy system, the identified objectives and the existing tradeoffs. Experiments confirmed that the DE algorithm was well adapted to the problem, where values obtained by the LP model were reproduced. The results confirmed that the DE algorithm reached high quality numerical solutions, since solutions reached by the LP model objectives were approached. Beyond the limitations our simulation may have, the results suggest that increases stocking rate can potentially result in economic benefit. At the same time, it is not necessary to reach the highest levels of milk production or herbage intake to improve the profits, since the solutions that maximize the margin over feeding cost are reached at suboptimal levels of milk production and herbage intake. Along this work, we have identified some areas that deserve further study. A first line of interest is to make a complete analysis from the computational point of view of the resource allocation model (which uses the DE algorithm) proposed in this article. The analysis should take into account both the quality of the solutions obtained and the computational cost of the algorithm. Additionally, it would be interesting to perform a configuration parameter analysis of DE in order to find the best parameter configuration for our resource allocation model.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Mathematical formulation

The problem was addressed and solved by a heuristic model, and was formulated as a mathematical programming model.
The definition of the parameters are presented in Table 5, and the decision variables were presented in Table 1.
The resulting mathematical formulation of the problem is shown in Eqs. (10) to (28).

Table 5
Parameters description.

Parameter	Description
S	Number of periods
B	Number of groups of cows
Z	Number of zones (pastures or feeding places)
T	Number of types of cows
MP	Milk price
CL_z, $z \in Z$	Calories level for each zone
BR_t, $t \in T$	Basal requirement for each type of cow
D_z, $z \in Z$	Distance from each zone to the milking parlour
BW_t, $t \in T$	Body weight for each type of cow
ENl	Energy per liter
DA_s, $s \in S$	Days per period (month)
C_t, $t \in T$	Number of cows of each type
PC_t, $t \in T$	Potential consumption for each type of cow
MG_{sz}, $s \in S$, $z \in Z$	Minimum level of herbage mass per hectare
F_z, $z \in Z$	Initial amount of food per hectare in each zone
RG_{sz}, $s \in S$, $z \in Z$	Rate of growth per day per hectare
H_z, $z \in Z$	Number of hectares per zone
$MinE_{st}$, $s \in S$, $t \in T$	Minimum energy consumption per cow
$MaxE_{st}$, $s \in S$, $t \in T$	Maximum energy consumption per cow
$MinP_{st}$, $s \in S$, $t \in T$	Minimum protein consumption per cow
$MaxP_{st}$, $s \in S$, $t \in T$	Maximum protein consumption per cow
$MinNDF_{st}$, $s \in S$, $t \in T$	Minimum NDF consumption per cow
$MaxNDF_{st}$, $s \in S$, $t \in T$	Maximum NDF consumption per cow
P_z, $z \in Z$	Amount of protein per kg of DM
NDF_z, $z \in Z$	Amount of neutral detergent fiber per kg of DM
$MinBS$	Minimum batch size
$MaxBS$	Maximum batch size

A.1. Objective functions

$$max \quad \frac{\sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T}((w_{sbzt} \times CL_z) - y_{sbz} \times x_{sbt}(BR_t/2 + Cte \times D_z \times BW_t))}{ENl} \tag{10}$$

$$max \quad PM \times \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T}\frac{((w_{sbzt} \times CL_z) - y_{sbz} \times x_{sbt}(BR_t/2 + Cte \times D_z \times BW_t))}{ENl} - w_{sbzt} \times RC_z \tag{11}$$

$$min \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt} \times RC_z \tag{12}$$

$$max \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt} \qquad \forall\, z \in P/Z = P + Sup \tag{13}$$

$$min \quad \sum_{s\in S}\sum_{b\in B}\sum_{z\in Z}\sum_{t\in T} w_{sbzt} \qquad \forall\, z \in Sup/Z = P + Sup \tag{14}$$

sa:

A.2. Resource allocation constraints

$$\sum_{z\in Z} y_{sbz} = DA_s \times 2 \qquad \forall\, s \in S, \forall\, b \in B \tag{15}$$

$$\sum_{b\in B} x_{sbt} = C_t \qquad \forall\, s \in S, \forall\, t \in T \tag{16}$$

$$\sum_{t\in T} x_{sbt} \geq MinBS \qquad \forall\, s \in S, \forall\, b \in B \tag{17}$$

$$\sum_{t\in T} x_{sbt} \leq MaxBS \qquad \forall\, s \in S, \forall\, b \in B \tag{18}$$

$$y_{sbz} \times x_{sbt} \times \frac{PC_t}{2} \geq w_{sbzt} \qquad \forall\, s \in S, \forall\, b \in B, \forall\, z \in Z, \forall\, t \in T \tag{19}$$

$$\sum_{b\in B}\sum_{t\in T} w_{sbzt} \leq v_{sz} - MG_{sz} \times H_z \qquad \forall\, s \in S, \forall\, z \in Z \tag{20}$$

A.3. Available food constraints

$$v_{sz/s=March} = F_z \times H_z \qquad \forall\, z \in Z \tag{21}$$

$$v_{sz} = v_{s-1z} - \sum_{b\in B}\sum_{t\in T} w_{s-1bzt} + RG_{sz} \times DA_s \times H_z \qquad \forall\, s \in S/s > 1, \forall\, z \in Z \tag{22}$$

A.4. Biological constraints

$$\sum_{z\in z} w_{sbzt} \times CL_z \geq MinE_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{23}$$

$$\sum_{z\in z} w_{sbzt} \times CL_z \leq MaxE_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{24}$$

$$\sum_{z\in z} w_{sbzt} \times P_z \geq MinP_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{25}$$

$$\sum_{z\in z} w_{sbzt} \times P_z \leq MaxP_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{26}$$

$$\sum_{z\in z} w_{sbzt} \times NDF_z \geq MinNDF_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{27}$$

$$\sum_{z\in z} w_{sbzt} \times NDF_z \leq MaxNDF_{st} \times x_{sbt} \times DA_s \qquad \forall\, s \in S, \forall\, b \in B, \forall\, t \in T \tag{28}$$

In this model, each cow type was represented by the index t, and each zone was represented by the index z. To identify each group of cows the index b was added (in the set B). Finally, each period (month) was represented by the index s (in the set S).

As a consequence, x_{sbt} represents the number of cows for each type for each group, y_{sbz} represents the number of times each group is assigned to each zone, w_{sbzt} represents the total consumption of DM for each group in each zone, and v_{sz} represents the available resources in each zone. A minimum level of residual herbage mass per hectare to ensure an adequate growth of pastures is considered.

This model assumes that the food resources available are shared uniformly between the cows assigned to a zone, so it is enough to know the whole zone consumption of DM and it is not necessary to represent the DM consumption for each cow.

The objective functions presented from Eq. (10) to Eq. (14) are the maximization of the milk production, the margin over feeding cost, the herbage intake and the minimization of the cost and the supplement intake.

The restrictions shown from Eq. (15) to (20) represent the resources allocation constraints. The restriction shown in Eq. (15) forces the sum of assignments for each zone to be equal to the number of days multiplied by 2. The restriction shown in Eq. (16) ensures that for each type of cows, the sum of cows for each group must be equal to the number of cows of that type. The restrictions shown in Eqs. (17) and (18) ensure the number of cows in each group to be bigger than the minimum group size and smaller than the maximum group size. The restriction in Eq. (19) forces the real consumption to be equal or lower than the potential consumption. Finally, Eq. (20) ensures the real consumption for each zone must be lower or equal than the available amount of food minus the minimum requirement for a controlled regrowth of the pasture.

Eqs. (21) and (22) represent the constraints related to food availability. In particular, Eq. (21) determines the available food in each period, which is calculated considering the food consumption in the previous period and the growth rate per day.

Eqs. (23)–(28) represent the biological constraints. These constraints are the same as those used in the LP model. Eqs. (23) and (24) force the minimum and maximum criteria for energy consumption to be respected. Eqs. (25) and (26) force the minimum and maximum criteria for protein consumption to be respected. Finally, Eqs. (27) and (28) ensure the minimum and maximum criteria for NDF consumption to be respected.

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